



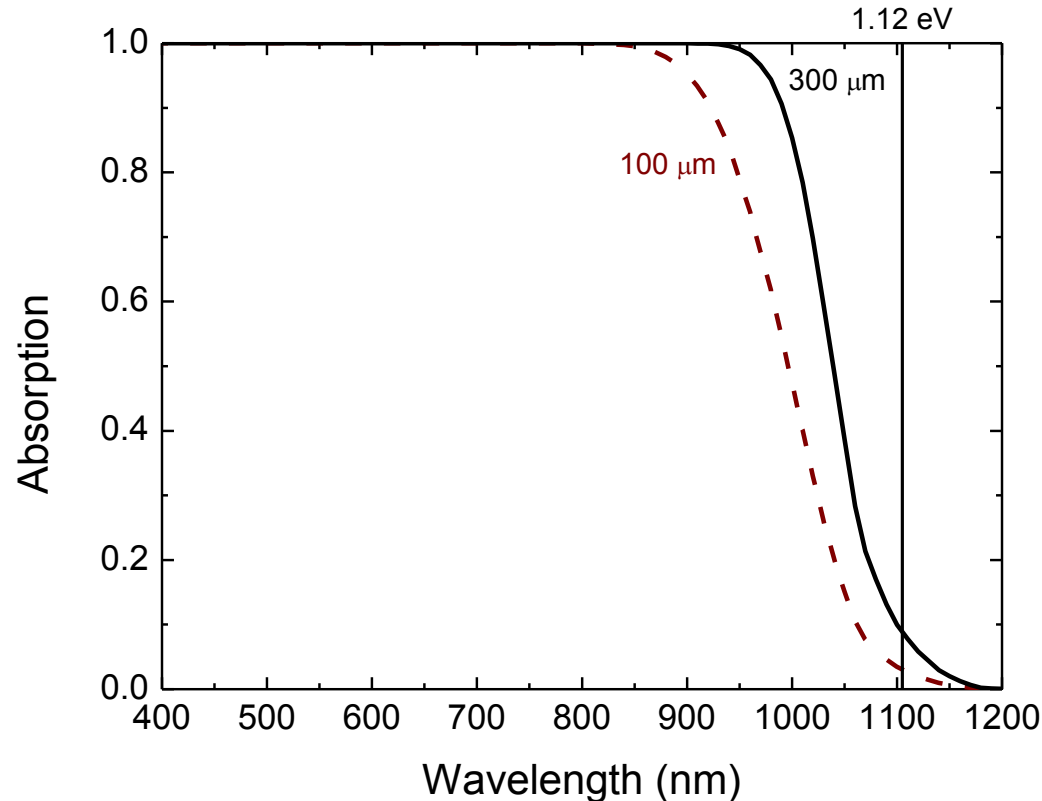
Understanding the mechanism of light-trapping

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Why light trapping

The direct gap makes silicon a weak absorber



Single pass absorption through d :

$$A = 1 - e^{-\alpha d}$$

$$\approx \alpha d \text{ (weakly absorbing region)}$$

Assuming:

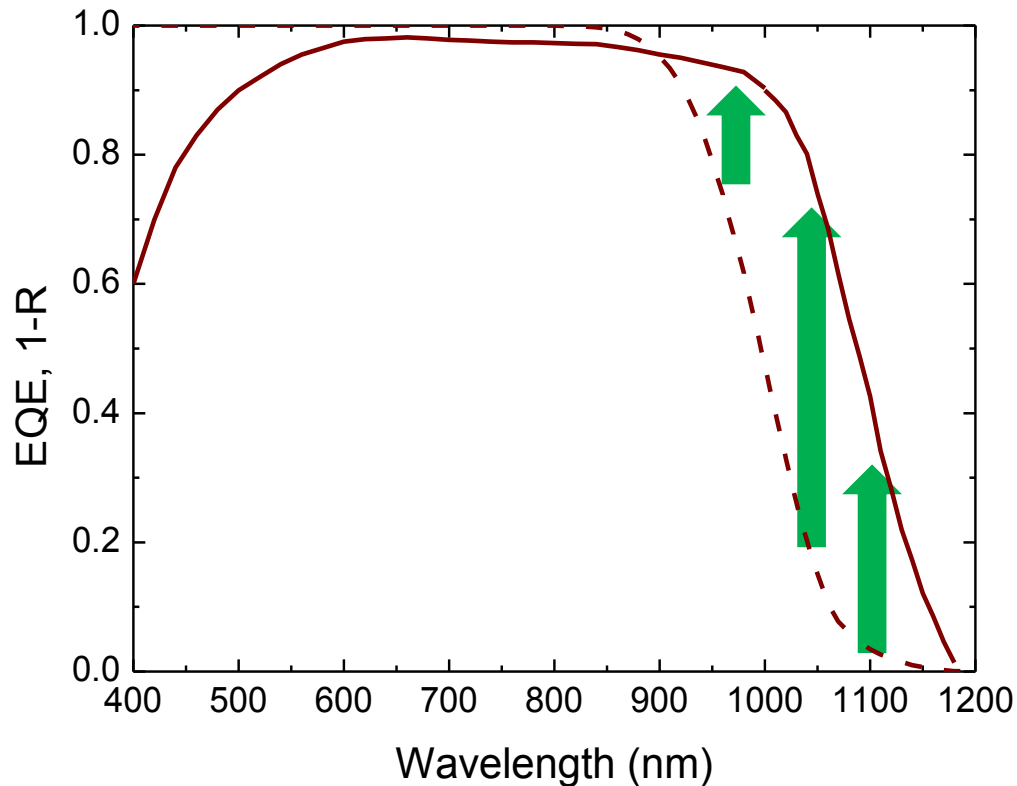
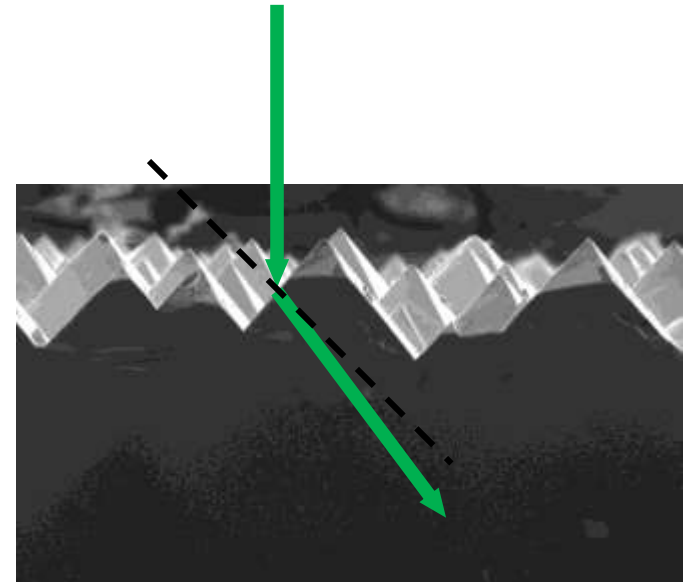
- no parasitic absorption
- perfect AR coating

300 μm c-Si wafers are not thick enough to absorb all light with $\lambda > 1000$ nm

100 μm c-Si wafers likely to become standard

Workaround: interface texture

EQE of 100 μm thick HIT solar cell



first described: Redfield, APL (1974)
recent exp. data: Holman, JAP (2013)

Theoretical limit of enhancement

Enhanced absorption by random scattering at texture (Deckman)

- ray tracing approach
- scattering prolongs the light path x2 (Lambertian)
- no parasitic absorption of BR ($\delta = 0$)
- no primary reflection

$$A = \frac{1 - e^{-4\alpha d}}{1 - e^{-4\alpha d} + \left(\frac{1}{n^2}\right)e^{-4\alpha d}}$$

Approximation for very weak absorption ($4(n^2 - 1)\alpha d \ll 1$):

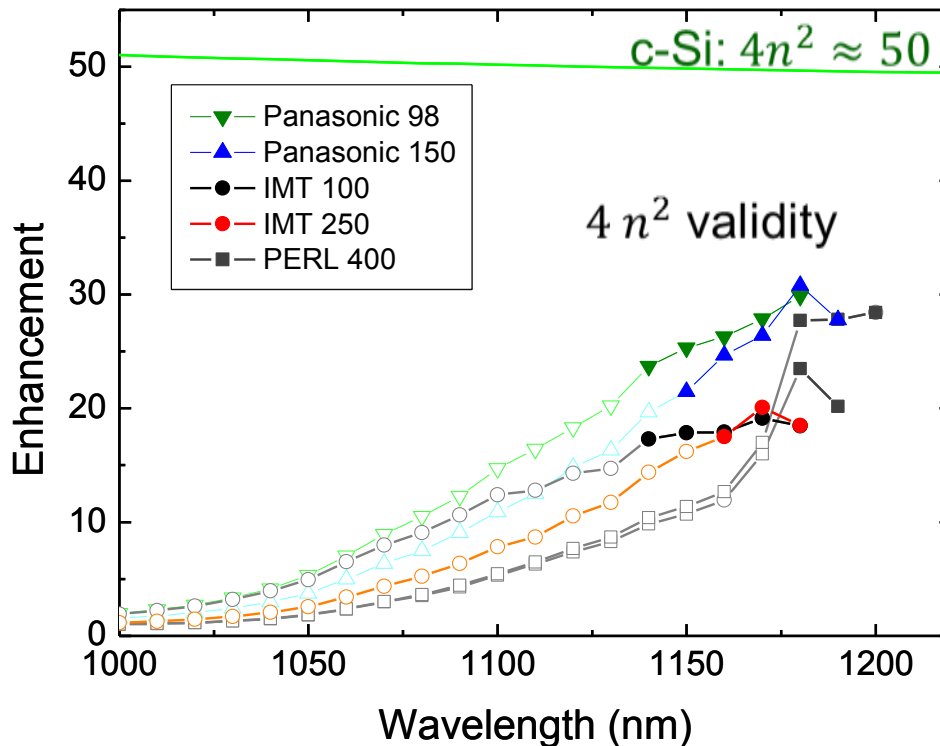
$$A = \underbrace{4n^2}_F \cdot \alpha d$$

=> same F as ergodic limit (Yablonovitch)

Yablonovitch, TED (1982)

Deckman, APL (1983)

How do actual devices perform?



estimation of F from experiment:

$$F = EQE/\alpha d \approx 20 \dots 30$$

Difference due to

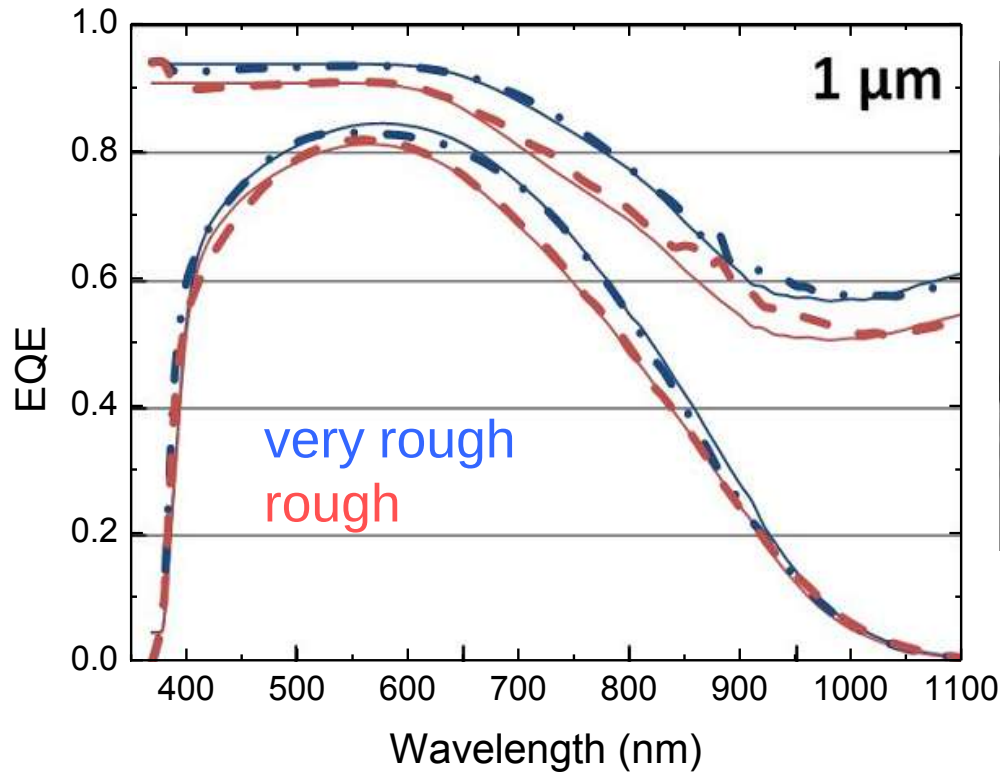
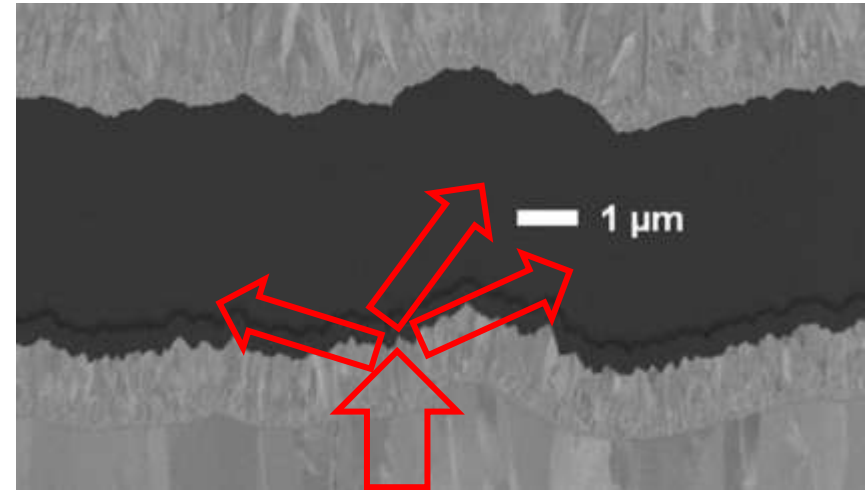
- incomplete randomization
- parasitic absorption
- reflection (better $F' = IQE/\alpha d$?)

Taguchi, JPV (2014)

Holman, JAP (2013)

Green, TED (1999)

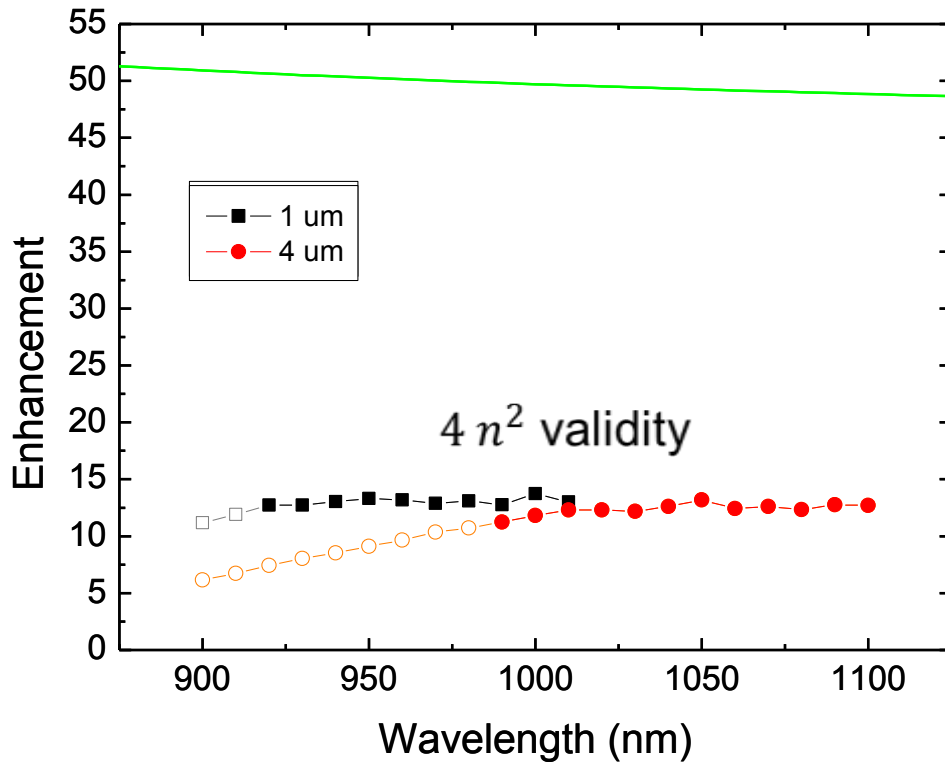
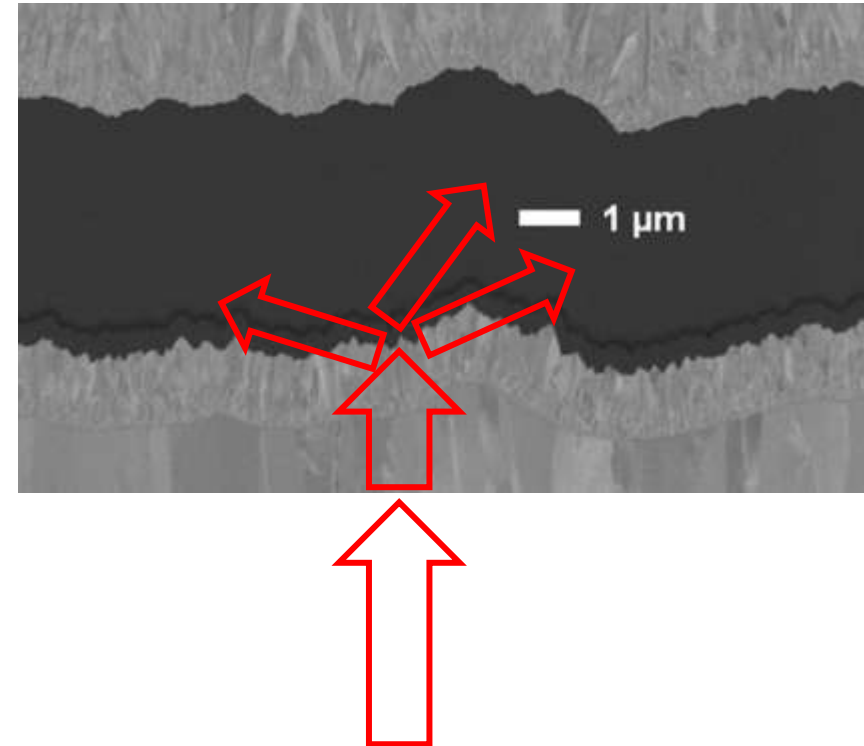
Required in thinner cells: smaller textures



Texturing on nano-scale
“Thick” absorber still described by ray tracing

first described: Deckman, APL (1983)
theoretical limit: Stuart, JOSA (1997)
data and model: Boccard, APL (2013)

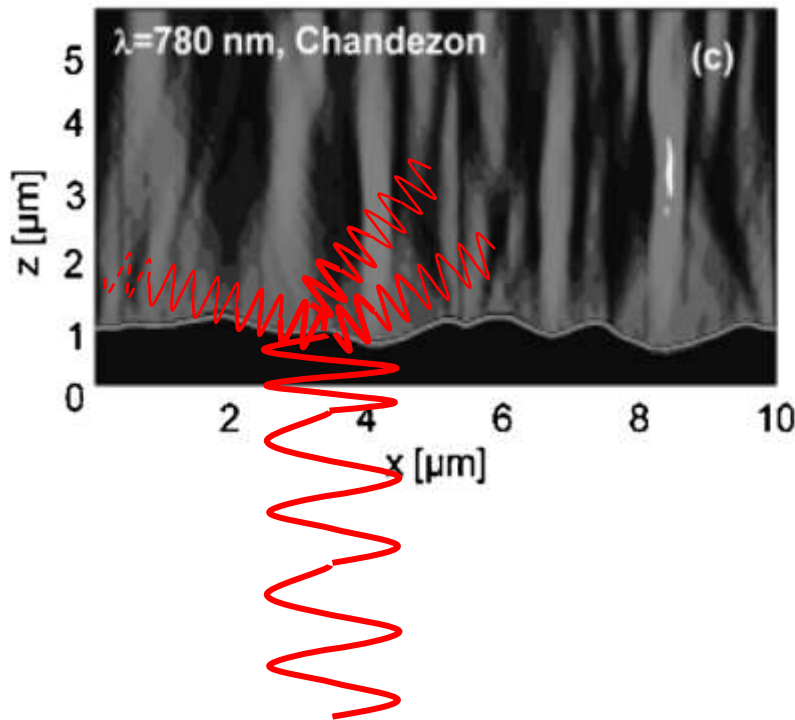
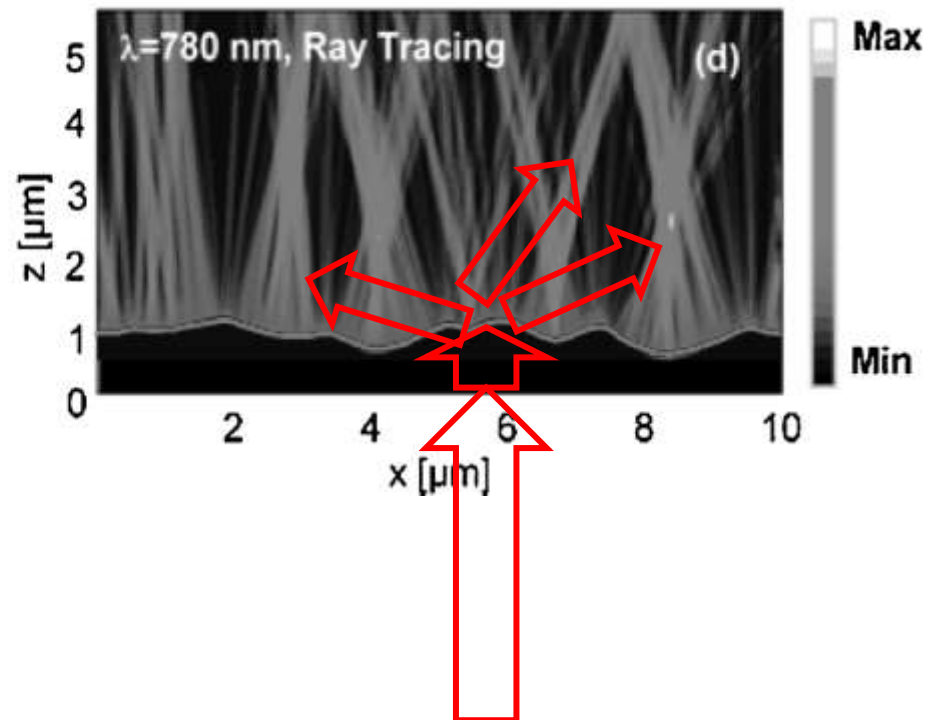
Enhancement in thin cells



Experimental enhancement: $F \approx 10 \dots 15$
reflection, incomplete randomization (as before)
parasitic absorption (more significant w.r. to thin absorber!)

data: Boccard, APL (2013)

Wave optics vs. ray tracing



Ray optics should work reasonably well for nc-Si cells (1...4 μm)

Schulte, PPV (2011)

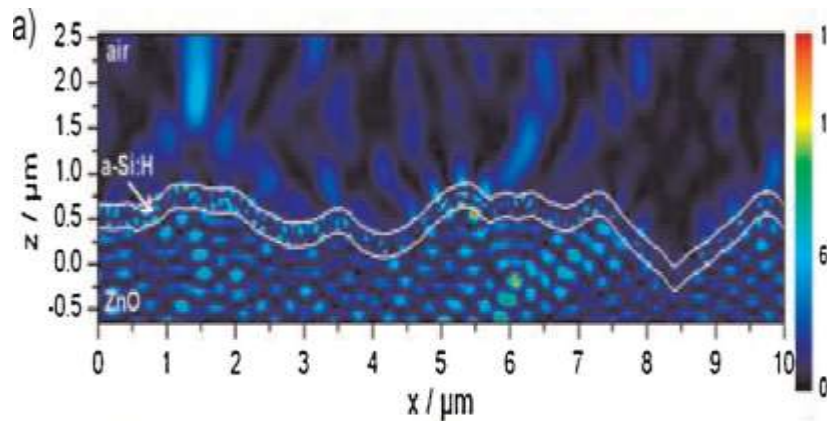
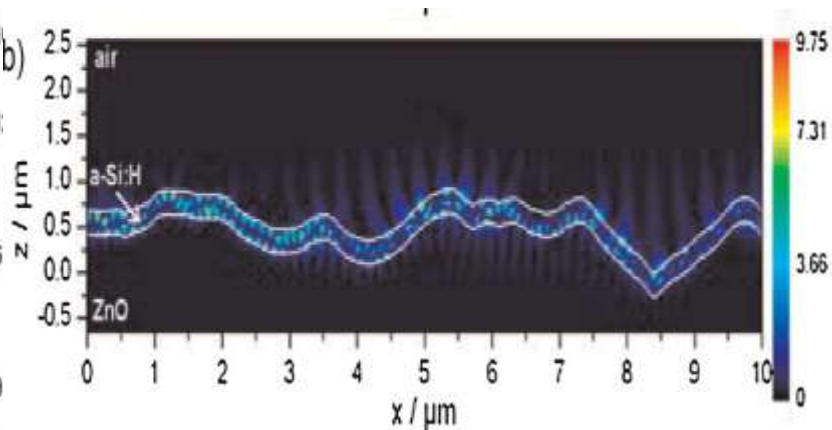
Near-field effects (wave optics)

Field intensity

$$E = \sum_k a_k e^{ikx} + c. c.$$

Field intensity bound to Si

$$E = \sum_{k > k_{\text{ZnO}}} a_k e^{ikx} + c. c.$$



@ $\lambda = 780 \text{ nm}$

Near field effects “leak” out of Si on a scale of ca. 500 nm

Bittkau, PSS (2010)

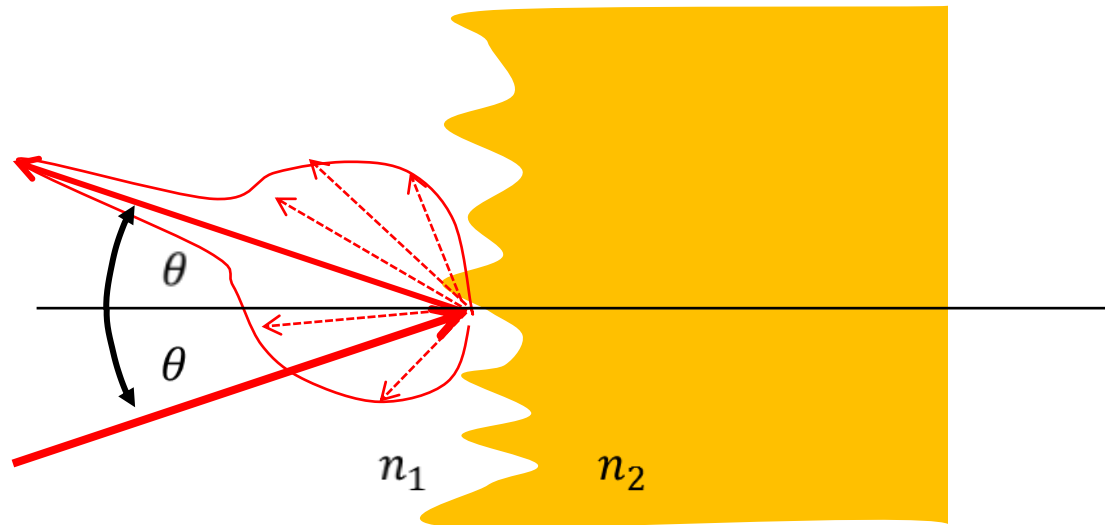
Describing light scattering textures with simplified models

Scalar scattering theory

Historically: interpretation of radar reflection from natural surfaces (buildings, ocean waves, etc.)

=> interest only in the specular reflection R_{spec} :

$$R_{spec} = R_T \exp\{-(4\pi/\lambda \cdot \sigma_{rms} \cdot n_1 \cos \theta)^2\}$$



Rice, Comm. Pure Appl. Math. (1951)

Davies Proc. Inst. El. Eng. (1954)

Scalar scattering theory

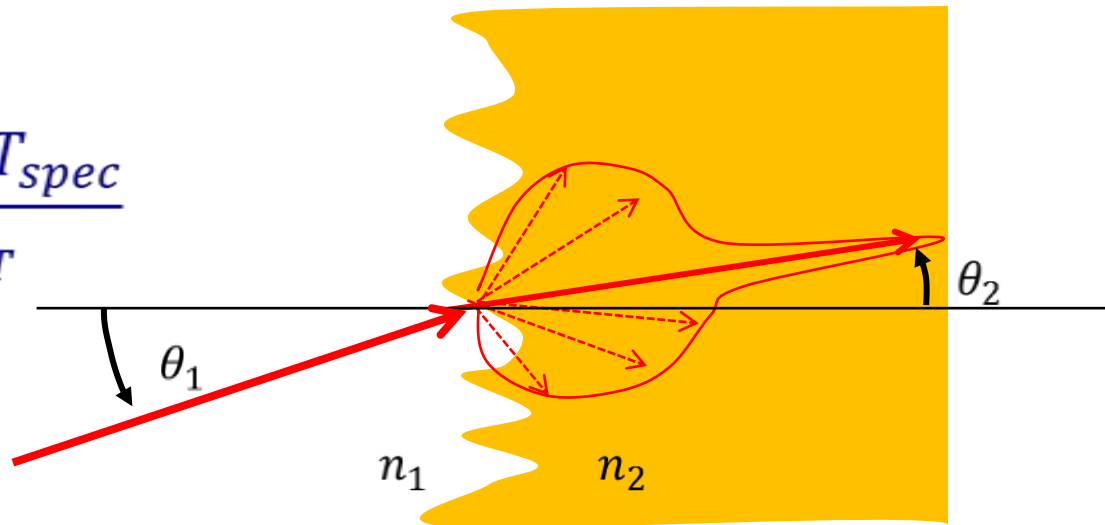
Formulation for transmission including refraction:

$$T_{spec} = T_T \exp\{-(2\pi/\lambda \cdot \sigma_{rms} \cdot |n_1 \cos \theta_1 - n_2 \cos \theta_2|)^2\}$$

Definition of Haze:

$$H_T = \frac{T_{diff}}{T_T} = \frac{T_T - T_{spec}}{T_T}$$

(T_T drops out)



Carniglia, Opt. Eng. (1979)

Scalar scattering theory

Empiric modifications:

$$T_{spec} = T_T \exp\left\{-(2\pi/\lambda \cdot \sigma_{rms} \cdot |n_1 \cos \theta_1 - n_2 \cos \theta_2|)^\delta\right\}$$

unknown for
internal interfaces

measurable by AFM,
in practice smaller σ_{eff}

Scalar theory: $\delta = 2$,
in practice empirically chosen

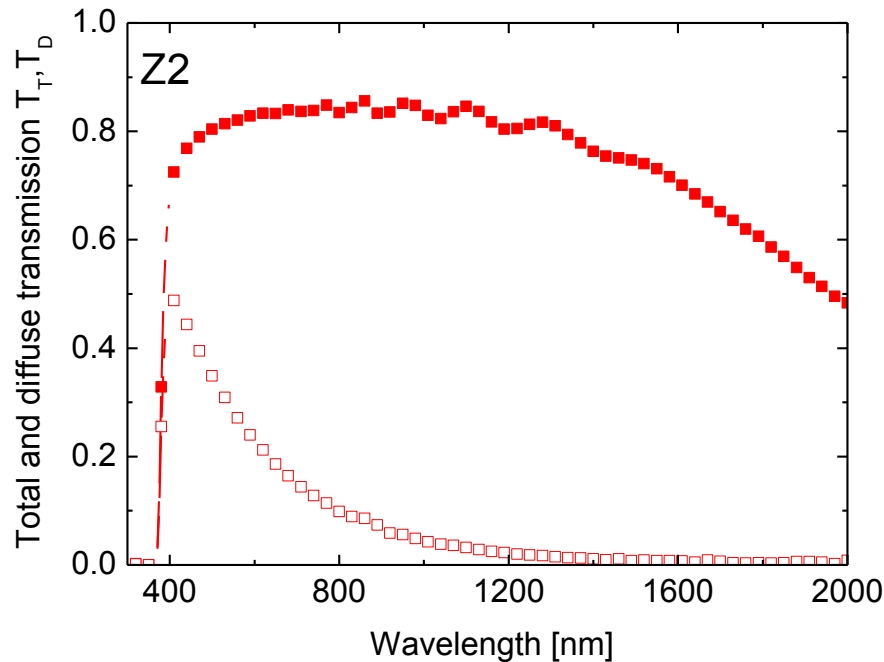
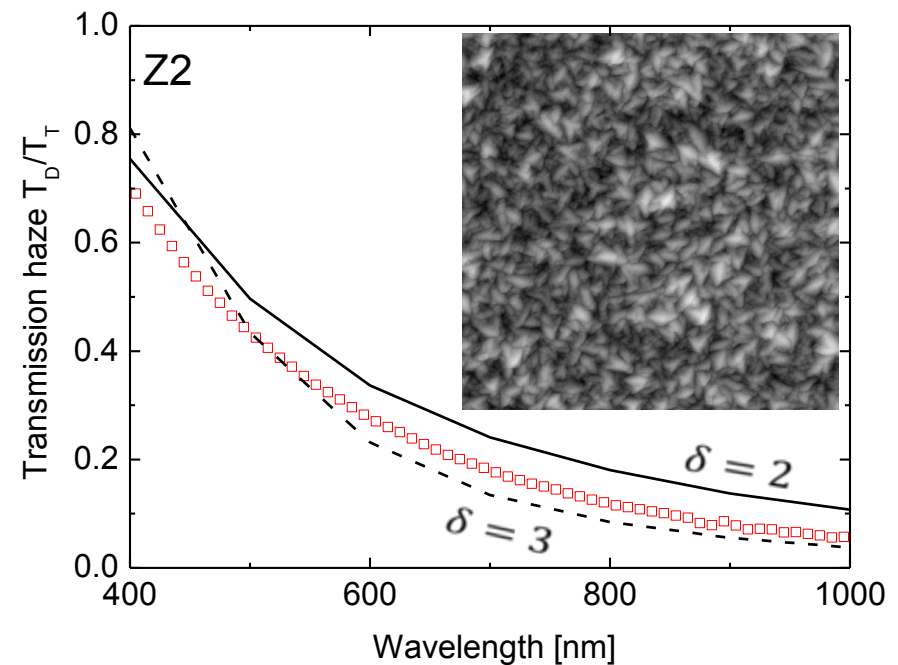
Angular dependence of diffuse light (ARS):
assumed, e.g. Lambertian

Used for modelling with e.g. ASA, SunShine, etc.

Zeman, JAP (2000)
Krc, SEM (2002)

Example: T and H_T of textured ZnO

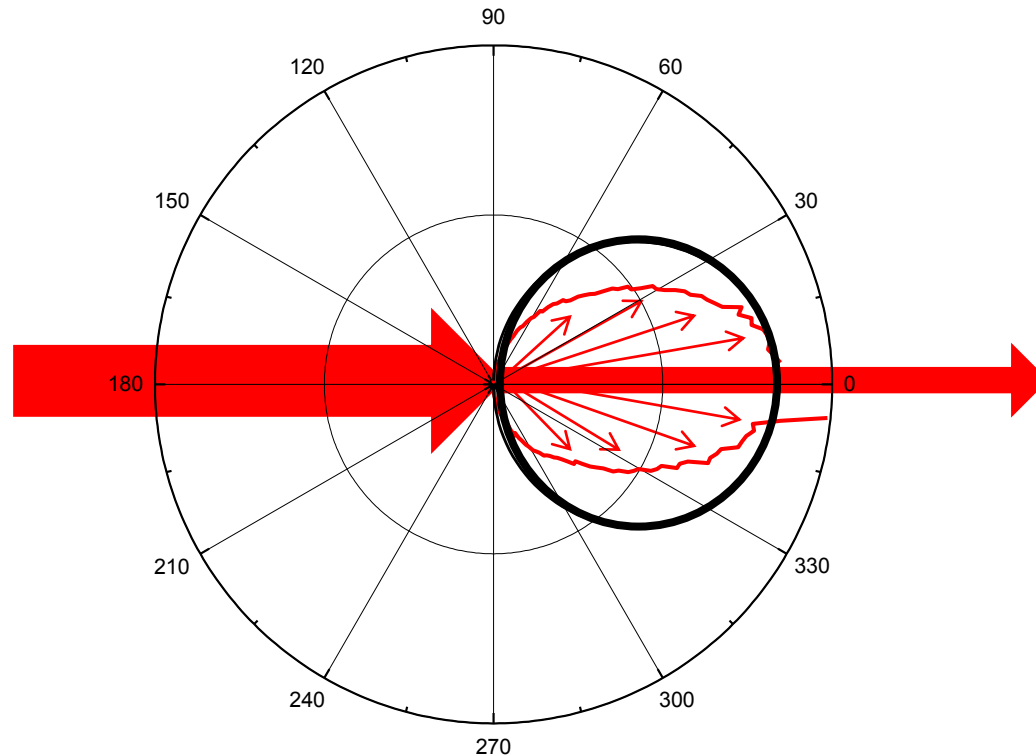
Scattering of 2 μm thick LP-CVD ZnO



- T_T changes with wavelength (UV: band gap, IR: FCA)
- H_T fits better with $\sigma_{eff} = 0.8 \cdot \sigma_{AFM}$ and $\delta = 2.7$

Issue: How do these parameters change for internal interfaces?

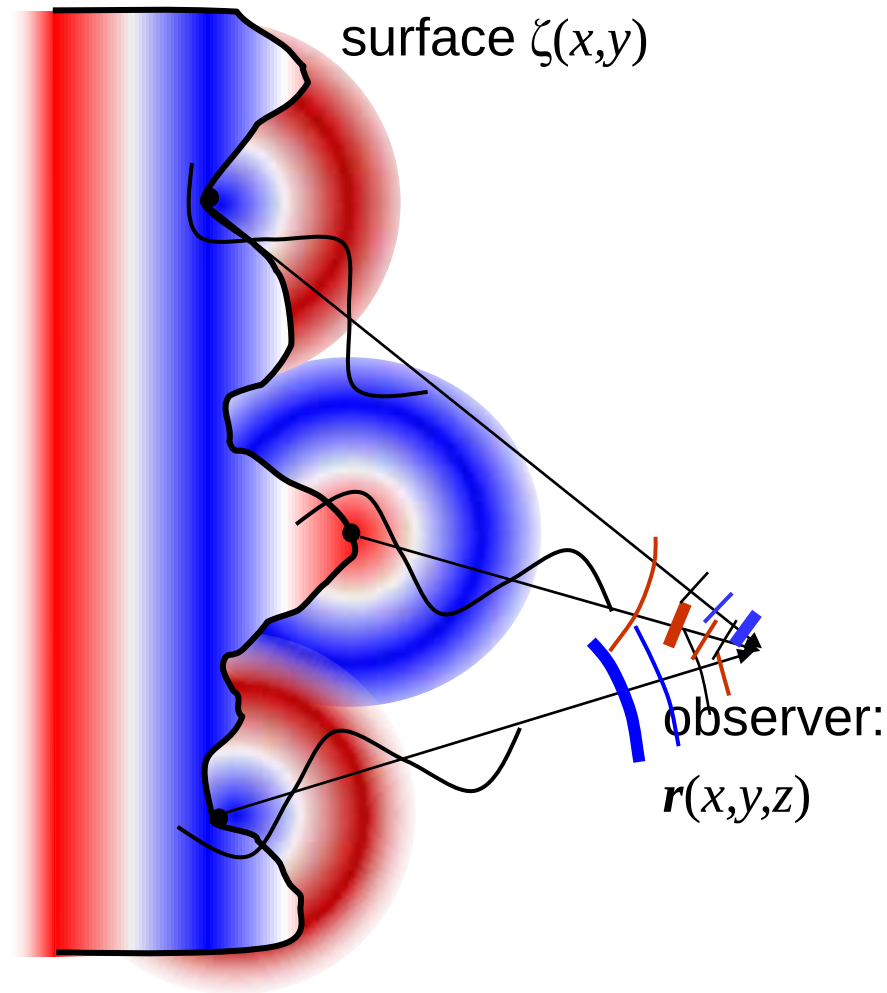
Example: *ARS* of textured ZnO



Scattering upon transmission into air (case for 543 nm)
Issue: How does the *ARS* change for internal interfaces?

(black: Lambertian scattering $\sim \cos \theta$)

Alternate modelling approach: Fourier theory



Huygens principle:

consider field at $\mathbf{r}(x,y,z)$ as superposition of spherical waves from each point of the surface $\zeta(x,y)$

Common choices of aperture function (pupil) $U_0(x,y)$:

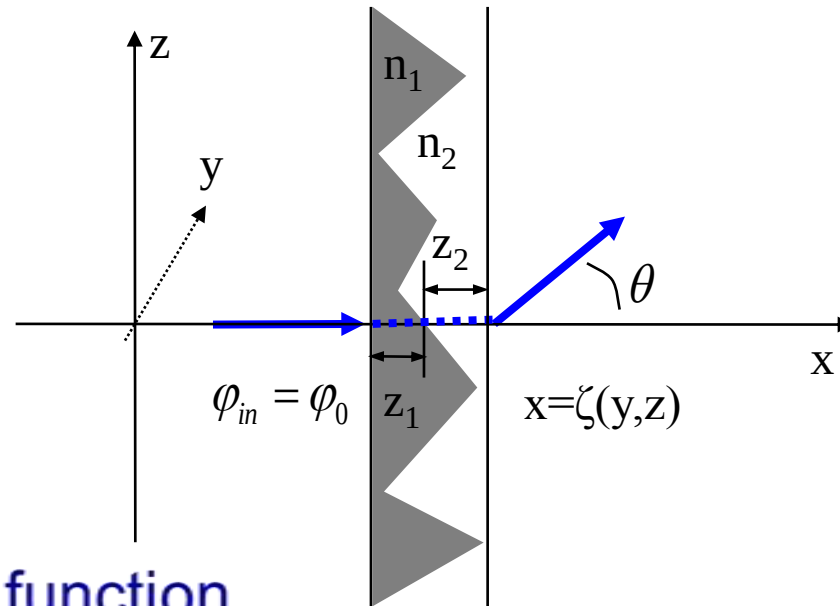
- **Amplitude modulation**
small (if any) absorption contrast within roughness zone!
(yields PSD, if used)
- **Phase modulation**
define the local phase w.r. to refractive index and depth

e.g. Goodman, Fourier Optics

Phase change

$$\Delta\varphi = kz_1 + kz_2 \quad (\text{with } z_1 + z_2 = d)$$

$$= 2\pi/\lambda \cdot \zeta(x, y) \cdot (n_1 - n_2) + \text{const}$$



Resulting pupil function
 $U(x, y) = \exp\{i \cdot \Delta\varphi\}$

Goodman, Fourier Optics
 Dominé, JAP (2010)
 Jäger, JAP (2012)

Fourier theory

Calculate transmitted **radiance** L by Fourier transform of U
(α, β : direction cosines; reflection accordingly):

$$L(\alpha, \beta) = \sqrt{T_T/A} \cdot \mathcal{F}\{U(x, y)\}$$

← remaining unknown parameter

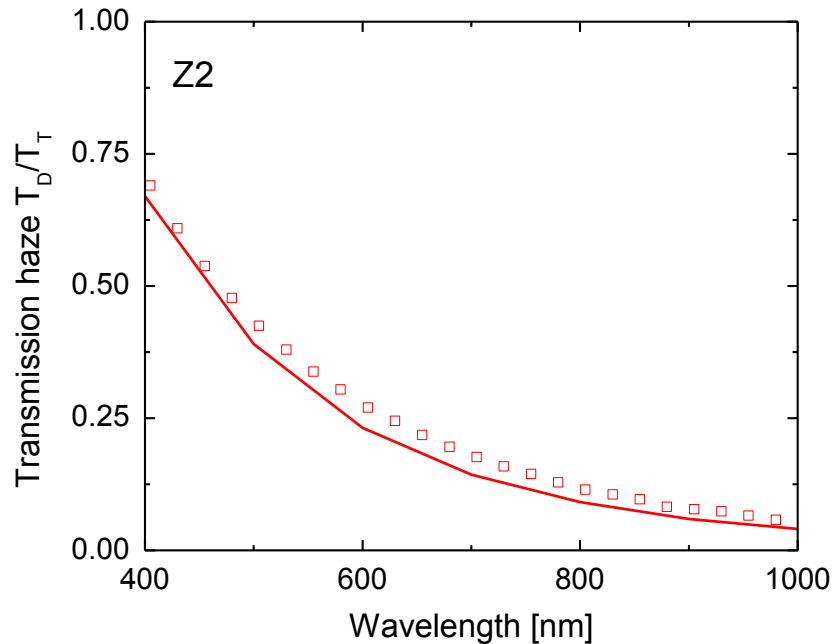
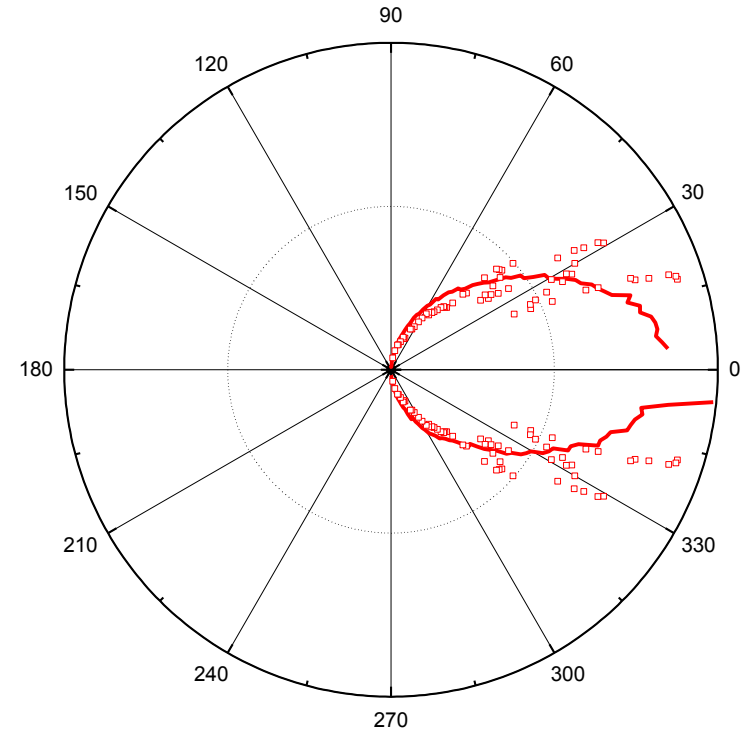
$$T_{spec} = |L(0,0)|^2$$

$$ARS(\theta) = |L(\alpha, \beta)|^2 \cdot \cos \theta$$

Used e.g. in new ASA version

Goodman, Fourier Optics
Dominé, JAP (2010)
Jäger, JAP (2012)

Haze and ARS of Fourier model

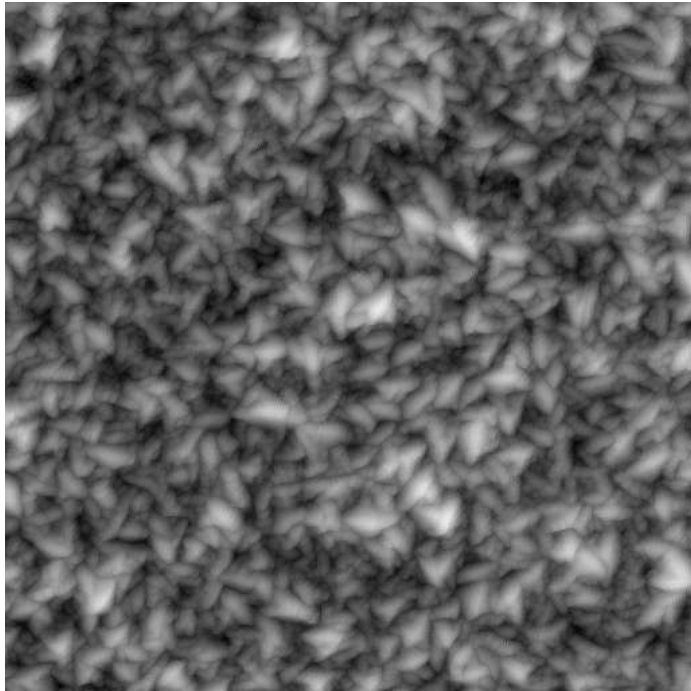


Both quantities predicted without any adjustable parameters!

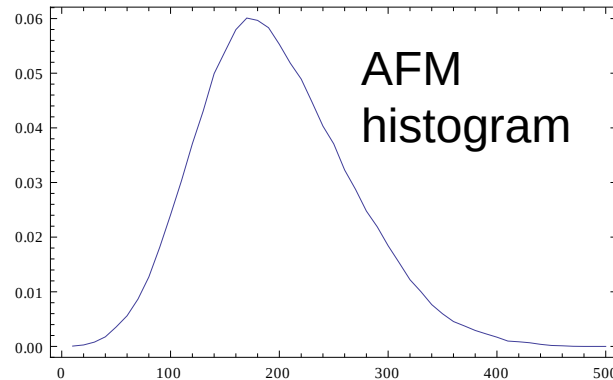
Total transmission T_T should still be quantified!

R/T for rough interface: use graded index

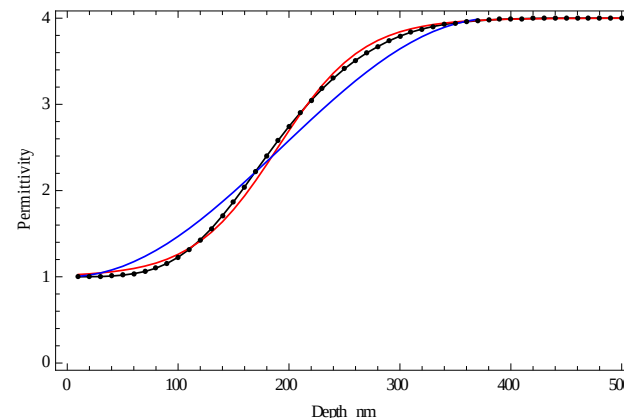
Example: LP-CVD ZnO (Z2)



$\sigma_{rms} = 68 \text{ nm}$
 $L_{cor} = 208 \text{ nm}$
 $Max(\zeta) - Min(\zeta) = 503 \text{ nm}$
(positively skewed, few very high tips)



- variation of $\rho(x) \sim$ integrated histogram (“rising sea level” argument)
- assume: $\epsilon(x) \sim \rho(x)$



on gradings and
effective medium theory:
Fujiwara, PRB (2000)

Theoretical treatment of graded index

Differential equation for field amplitude within grading

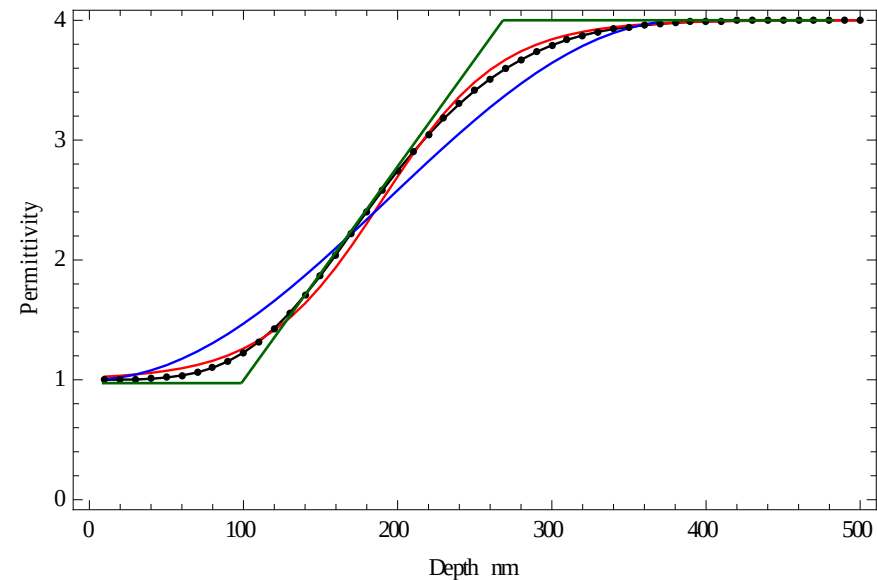
$$\frac{d^2}{dx^2} E(x) + \epsilon(x) k_0^2 \cdot E(x) = 0$$

Example of trial functions:

$$\epsilon(x) = n_1 + (n_2 - n_1) \left[1 + \tanh\left(\frac{x - x_0}{w}\right) \right] / 2$$

$$\epsilon(x) = n_1 + (n_2 - n_1) [1 - \cos(x \cdot \pi / w)] / 2$$

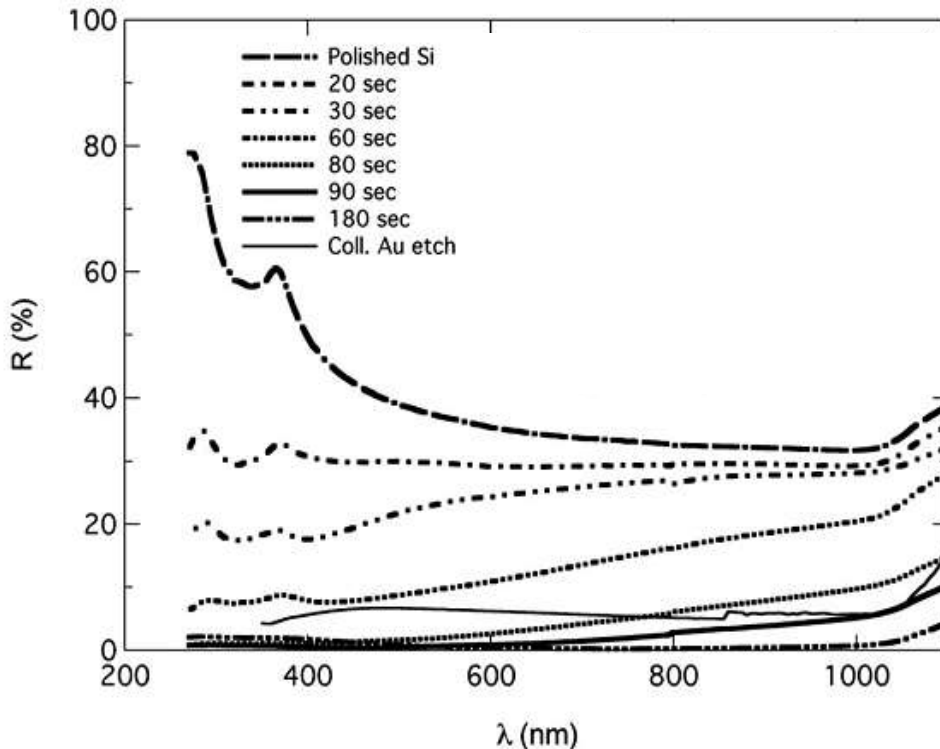
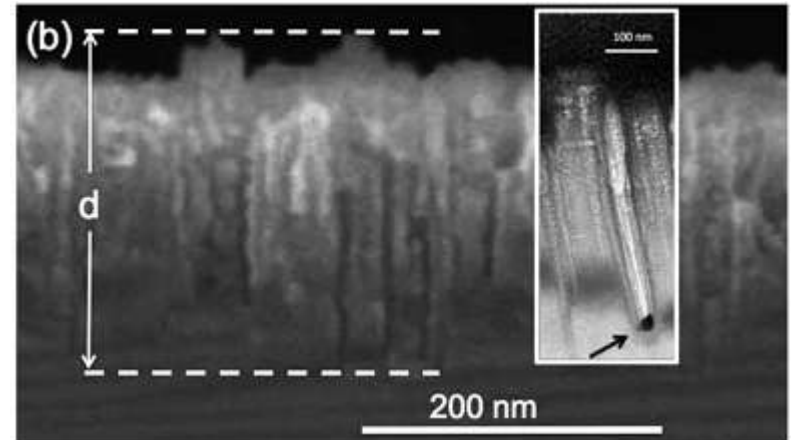
$$\epsilon(x) = n_1 + (n_2 - n_1) [x / w]$$



Note: only the linear grading has a (simple) analytic solution, $Ai(x)$ (Airy function)

Sheldon, JoSA (1982)

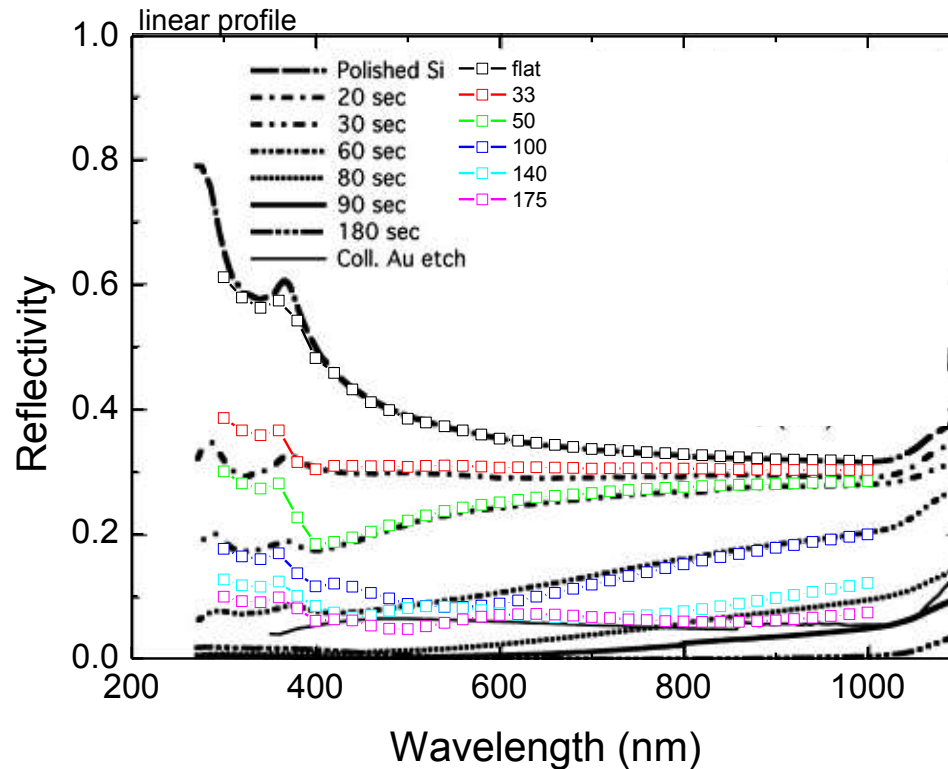
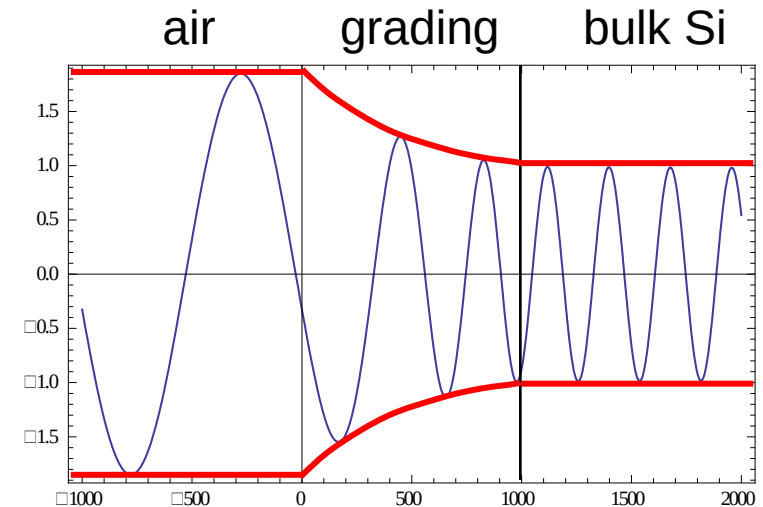
Example: reflection of “black” silicon



Reflection drastically reduced by rough surface zone
authors find linear grading of density

HAuCl_4 etch, Branz, APL (2009)

Result for linear grading



$$E_{air} \sim f \cdot e^{ik_0 x} + r \cdot e^{-ik_0 x}$$

$$E_{grading} \sim Ai(x)$$

$$E_{bulk} \sim e^{ik_{Si} x}$$

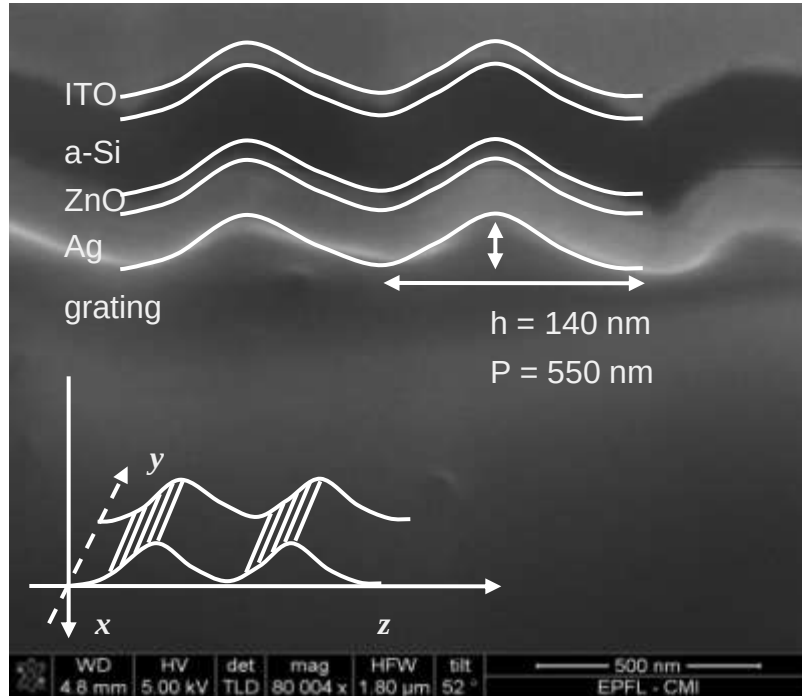
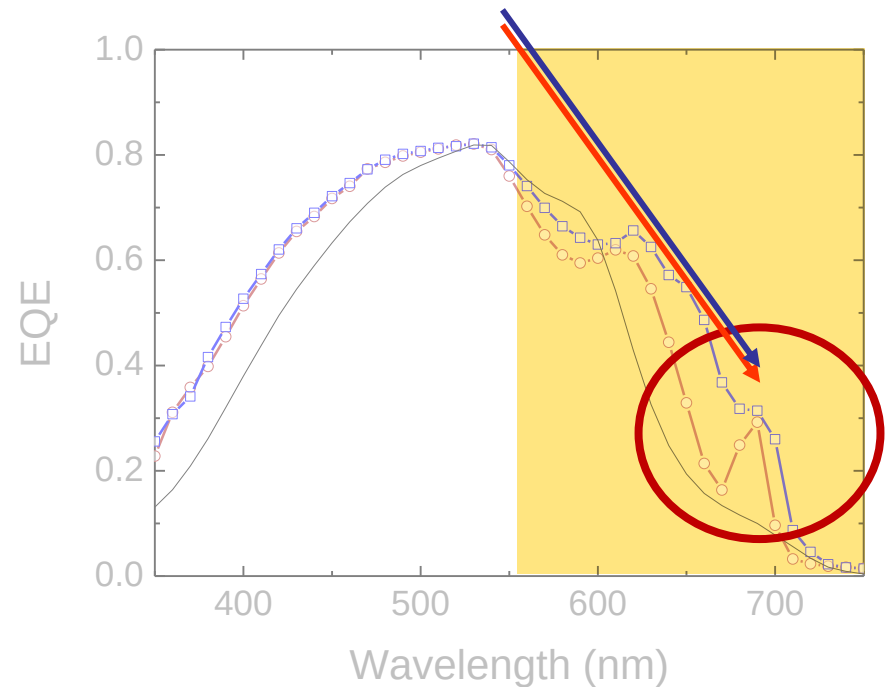
$$R = |r/f|^2$$

Solution fits reasonably for moderate depth here: up to ca 100 nm, but fitted depths are different from measured depths; cos- or tanh-gradings fit better

Sheldon, JoSA (1982)

Alternate approach:
simplify cell instead of model

Solar cell on sinusoidal 1D grating



Absorption enhancement by grating-coupling to guided modes
Resonances at 690 nm (corresponds to 1.8 eV)

K. Söderström, APL, (2010)

Resonances: explain with dispersion diagram

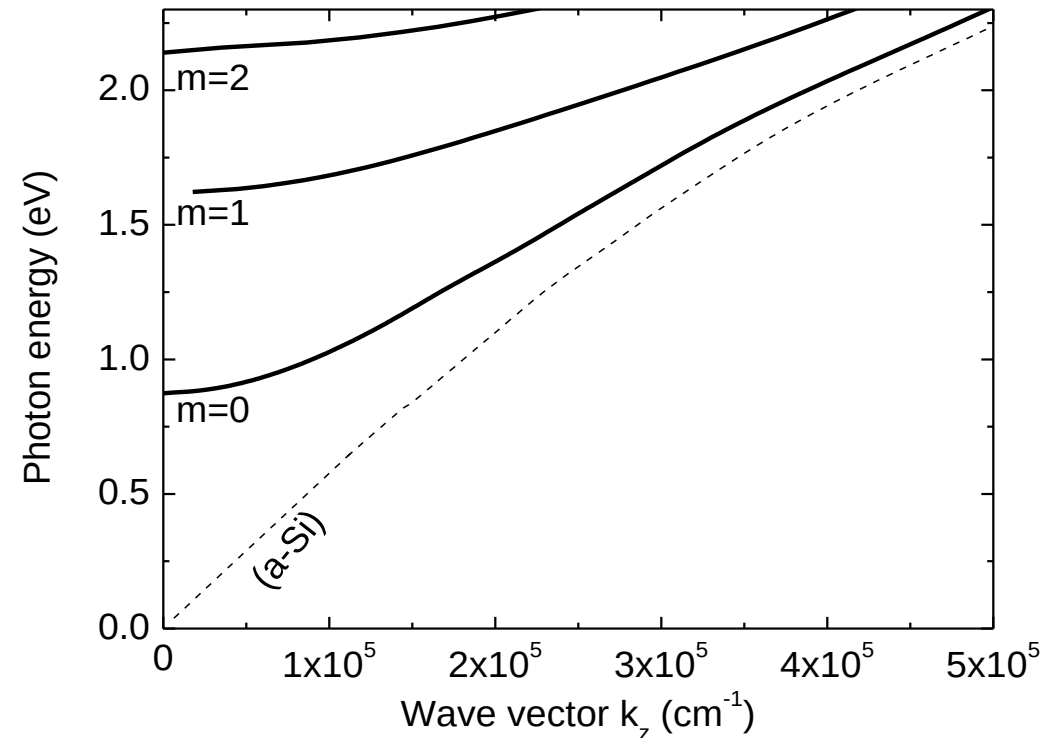


Illustration:

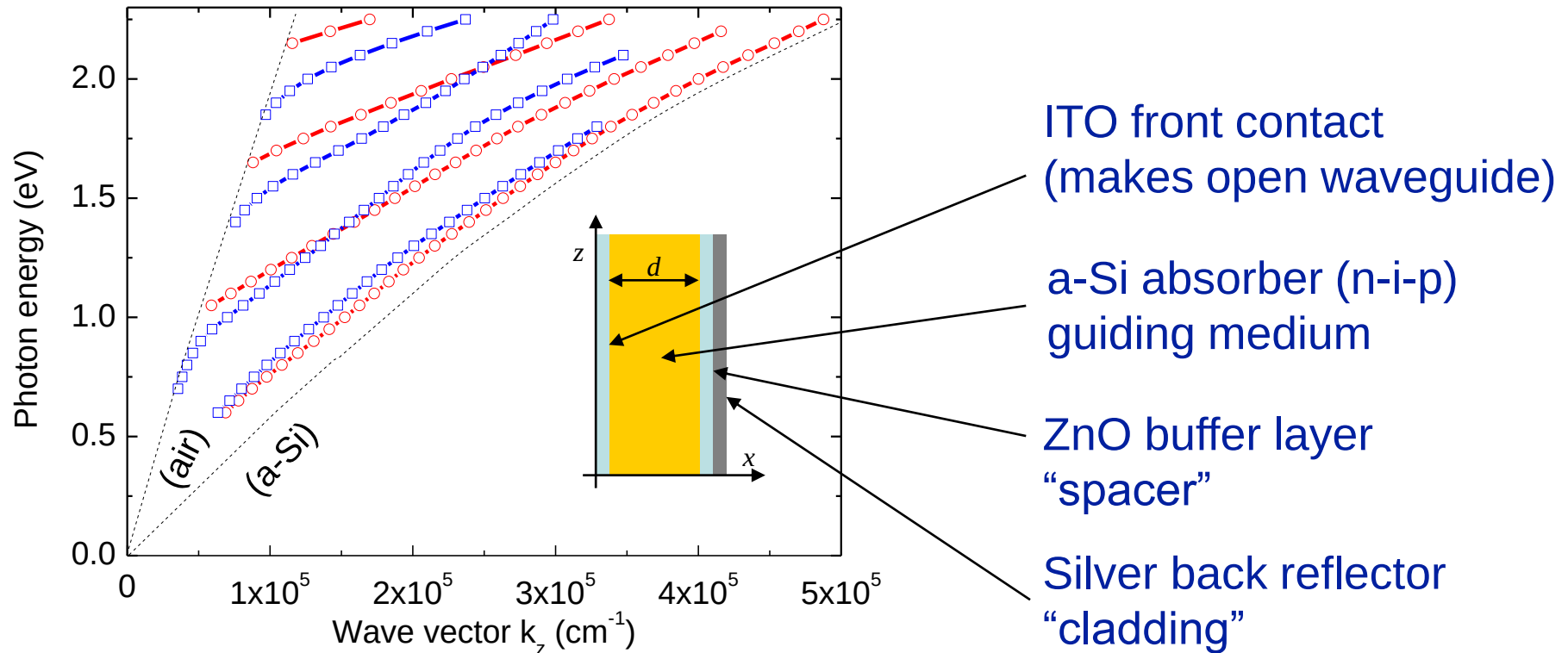
200 nm a-Si btw. perfect mirrors
 $\Rightarrow$ confine multiples of half
wavelength (nodes at boundaries)

Dispersion relation:

$$\begin{aligned}\omega(k) &= \frac{c}{n} |k| \\ &= \frac{c}{n} \sqrt{k_x^2 + k_z^2} \\ &= \frac{c}{n} \sqrt{((m+1) \cdot \pi / d)^2 + k_z^2}\end{aligned}$$

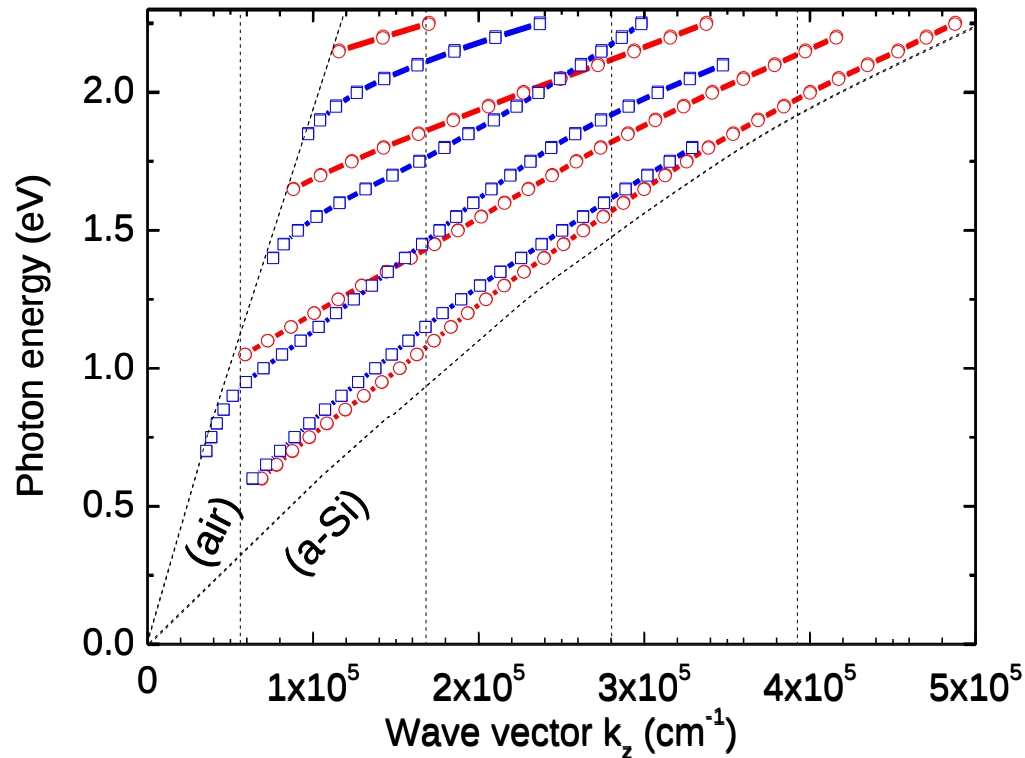
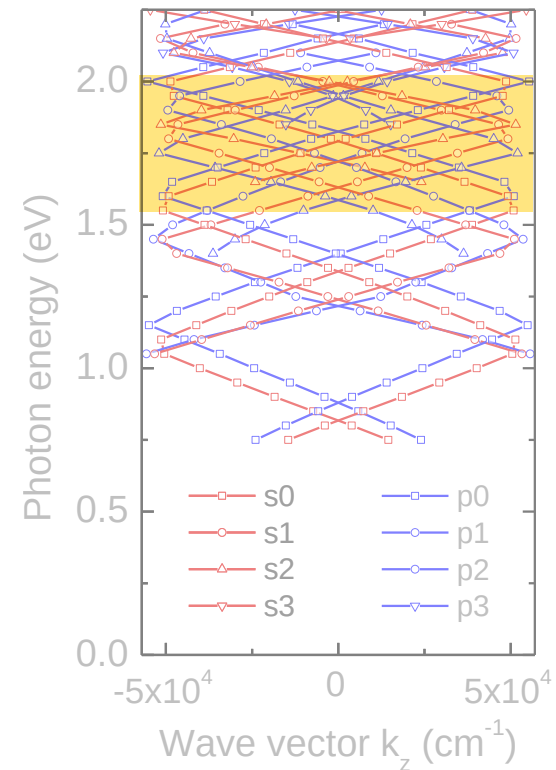
- symmetric w.r. to negative k_z (fwd and bwd travelling waves)
- cut-off at low energy: $d\omega/dk = 0$
- asymptotic behaviour: $\omega(k) = ck_z/n$ (light line of silicon)

Resonances: explain with dispersion diagram



Incident light ($\omega = ck_{||} = ck_0 \sin \theta$): line along the ordinate ($\theta = 0$).

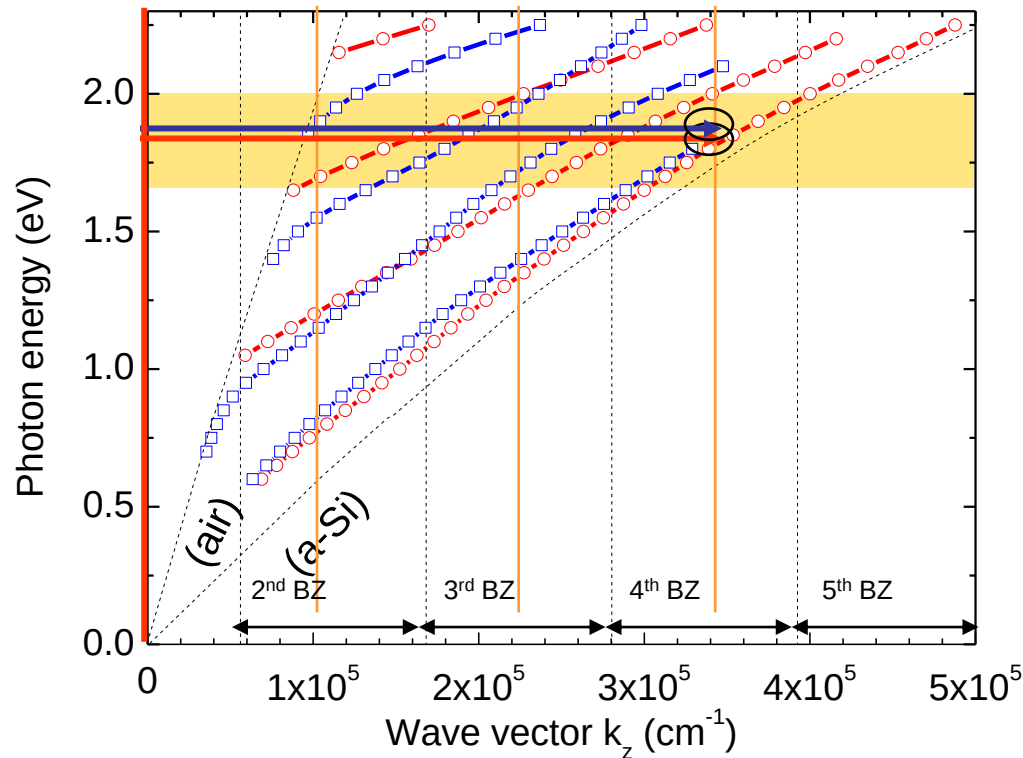
Coupling by periodic interface texture



Effect of periodicity:

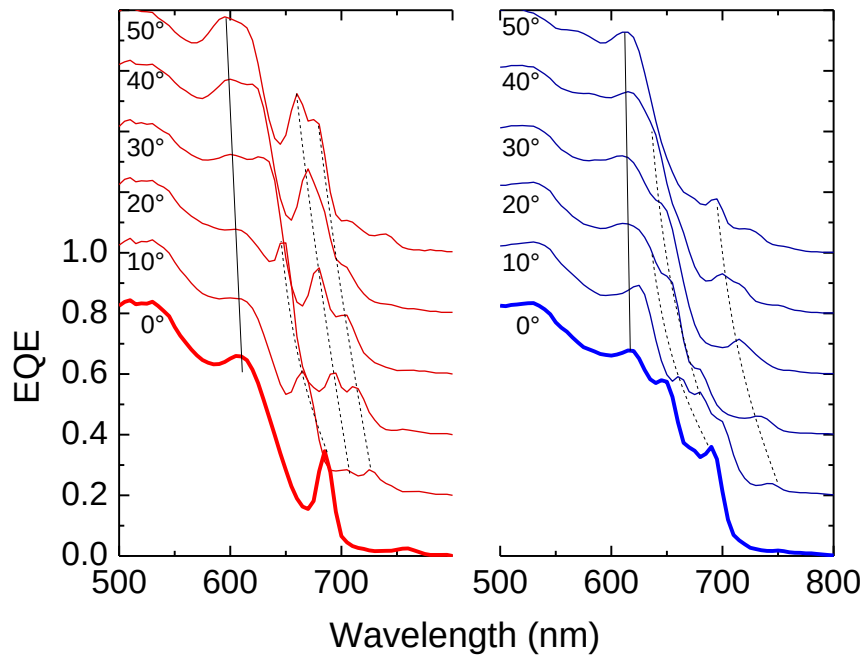
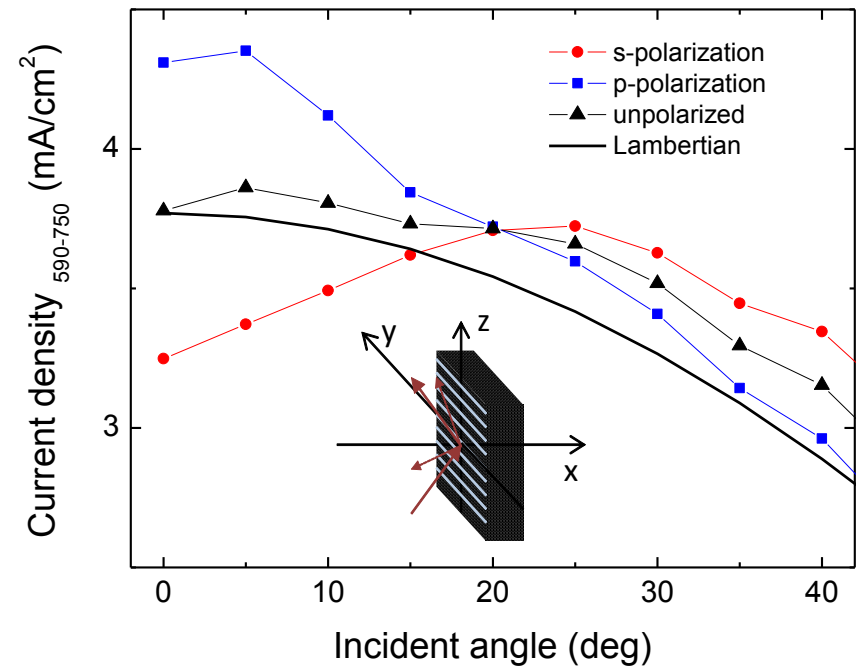
- Definition of Brillouin zones in dispersion diagram
- Reduced scheme

Coupling by periodic interface texture



Incident light (light line on abscissa) is translated into higher order BZs
Resonant coupling to s₀/p₀: expected at 1.8 eV (690 nm) in 4th BZ

Angular dependence of EQE and current

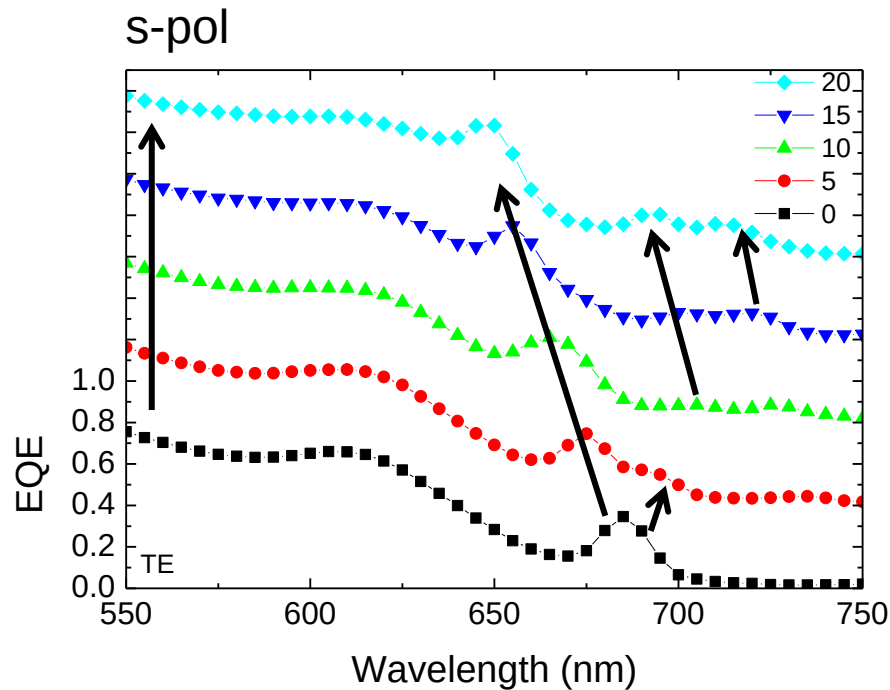
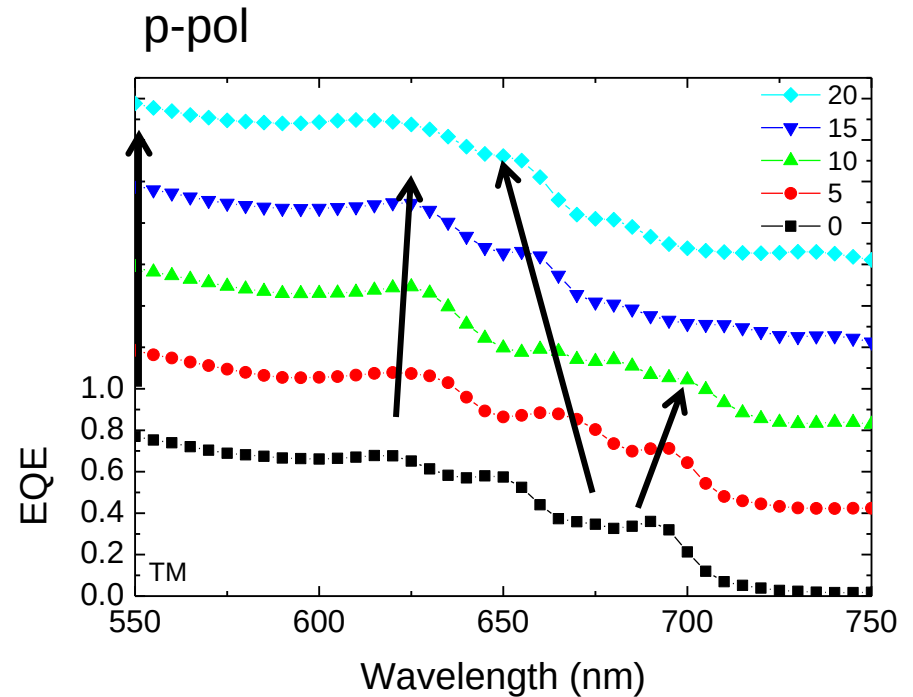


We find:

- little dispersion of Fabry-Perot resonances (high index of Si)
- strong dispersion of waveguide resonances
- highest photocurrents for $\theta \neq 0^\circ$
- unpolarized average higher than Lambertian cos-dependence

F.-J. Haug, APL, (2013)

Angular dispersion of resonances

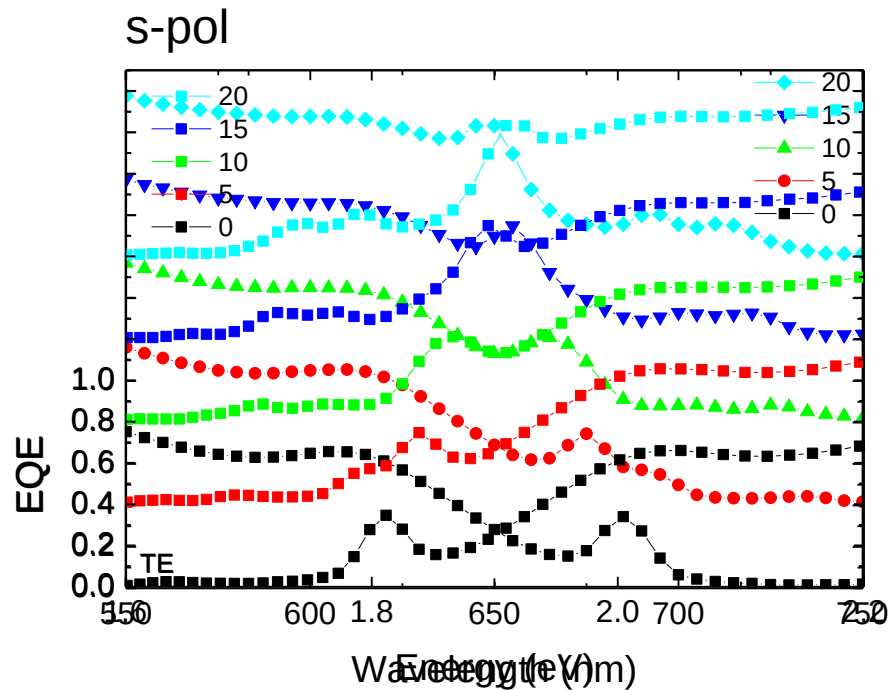
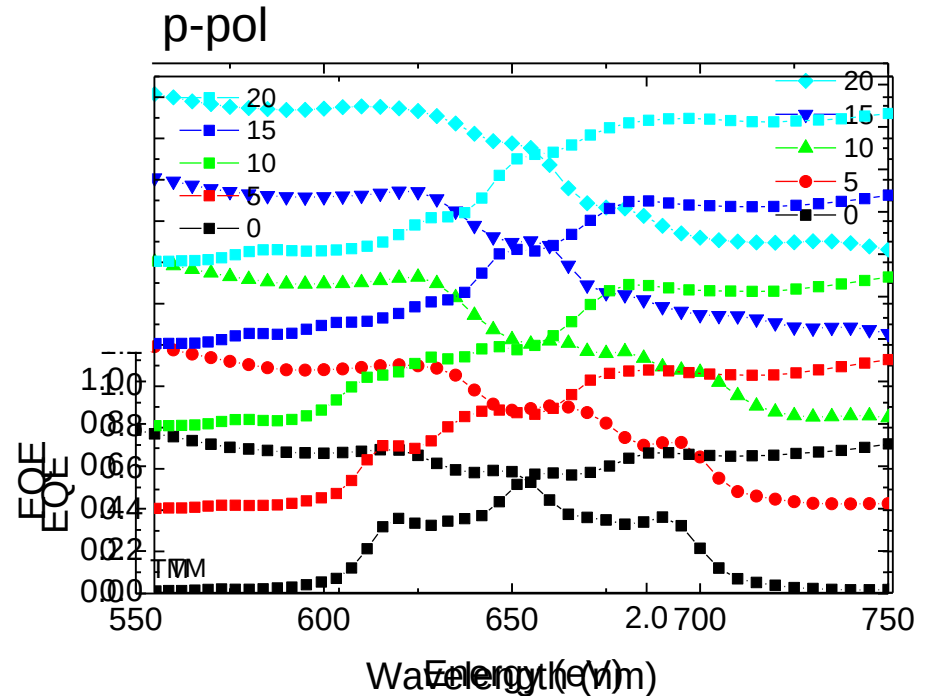


- little dispersion of Fabry-Perot resonances (high index of Si)
- strong dispersion of waveguide resonances (red and blue shift)

K. Söderström, APL, (2010)

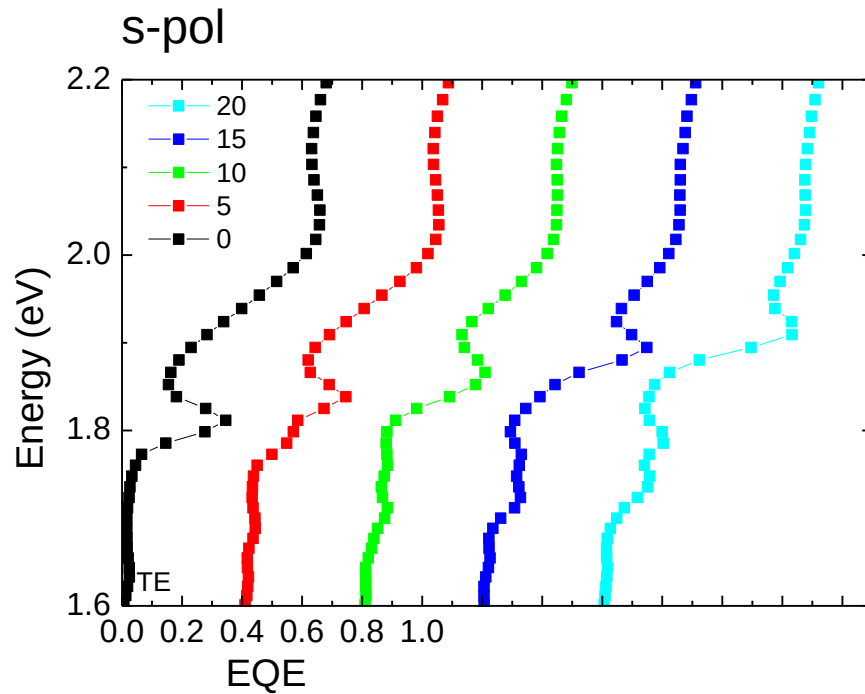
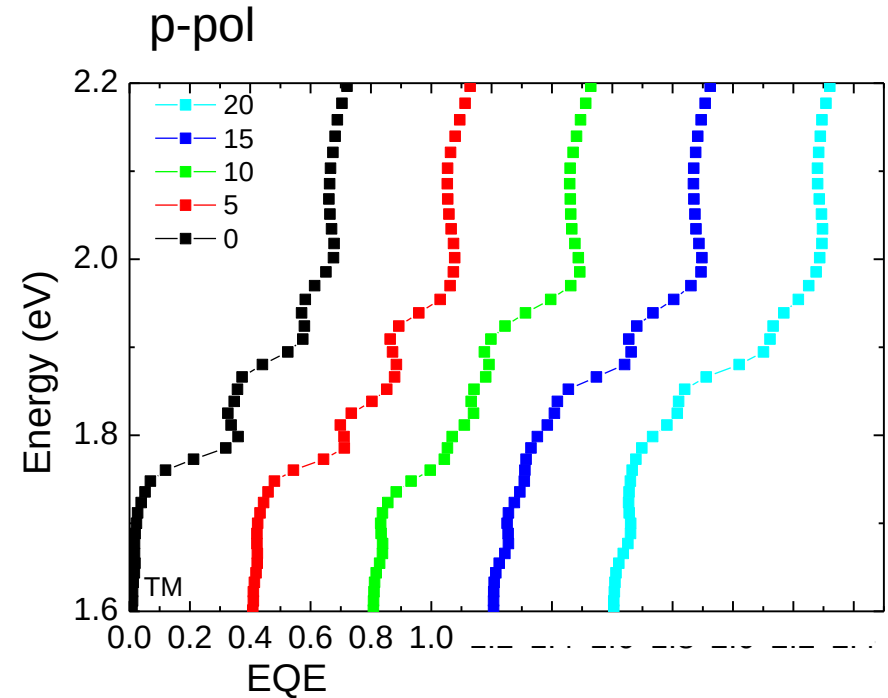
F.-J. Haug, JPV (2013)

Relation to dispersion diagram



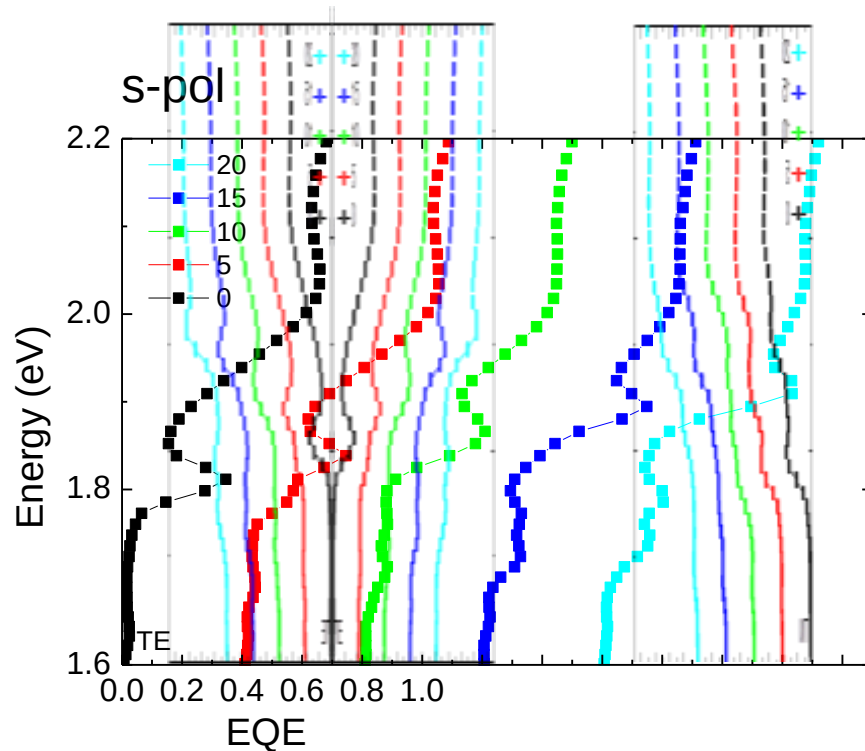
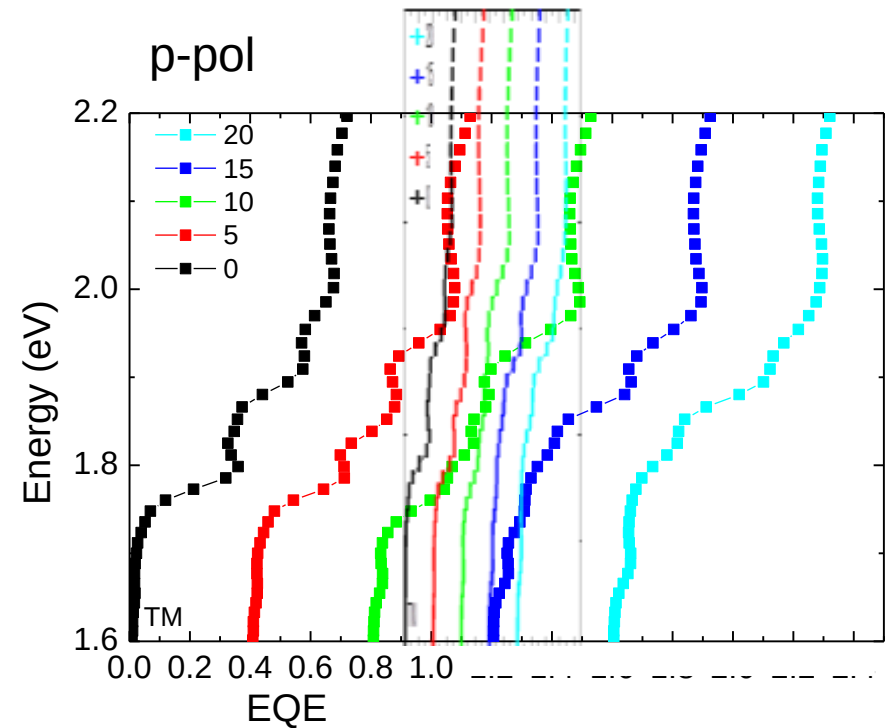
Plot with respect to energy: $E[h\nu] \triangleq 1241/\lambda[nm]$

Relation to dispersion diagram



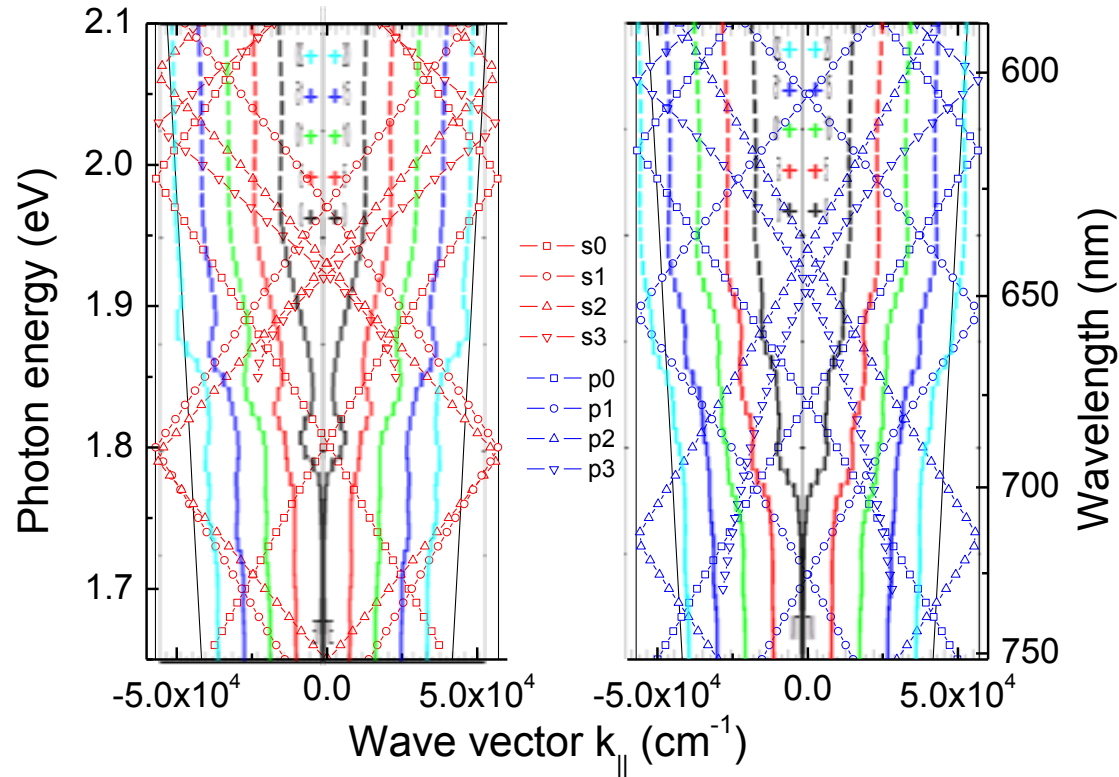
flip axes (energy upwards)

Relation to dispersion diagram

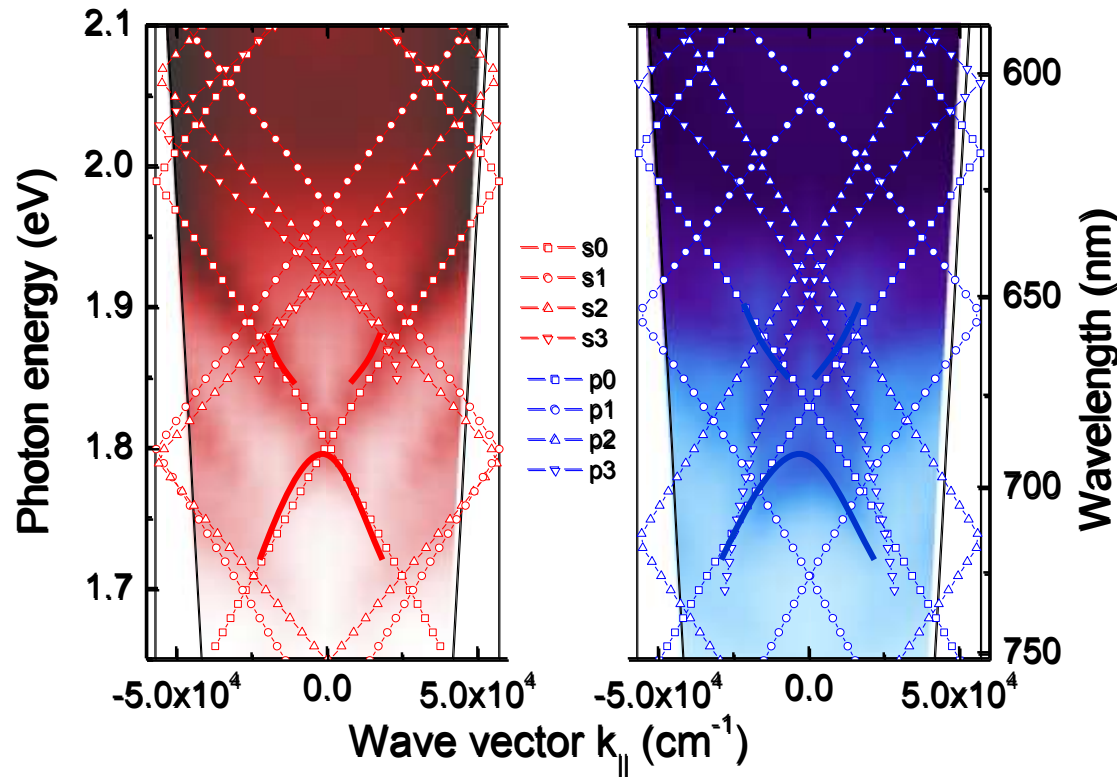


Map over reduced dispersion diagram

Relation to dispersion diagram



Formation of band gaps



Band gaps expected at points of crossing:

- for low mode order
- for high symmetry (e.g. centre of BZ)

- ☺ small photonic gaps found
- ☹ upper band disappears for 0° incidence
- => concept of photonic DOS may be misleading
- => reduced jsc for perpendicular incidence!

F.-J. Haug, APL, (2013)

Conclusions

- Photonic concepts have a long history in PV
- Modelling should be fast (sorry, FDTD, FEM etc.) but still accurate
 - reduce empiric parameters as much as possible
 - qualify predictions at intermediate steps
- What we also need:
 - absorbers with better material quality
 - innovative materials for the supporting layers (free carrier abs., etc)
 - consider customer's acceptance (PbS, CdSe, $(\text{NH}_3)\text{PbI}_3$, etc.)

Thank you for your attention

- **Acknowledgement for funding**
 - Swiss Federal Office for Energy
 - Swiss National Science Foundation
 - EU FP7 (FastTrack)
- **Thanks to all members of the PV-Lab**

